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Scale invariance and intermittency in a creep-slip model of earthquake faults

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The dynamics of a generalization of the one-dimensional, spatially discretized Burridge-Knopoff model (slider-block model) is investigated numerically. Plastic deformation of the fault interface is considered in addition to rigid sliding (creep-slip model). The event-size distribution exhibits scale invariance ($\beta=1.5$), as does the power spectral density of the intermittent time series of the spatially averaged sliding rate ($\sigma=1.3$). A diffusive cellular automaton model that reproduces the algebraic correlations in the event-size distribution in the presence of dissipation is proposed. [S1063-651X(99)50906-9]

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The suggestion by Bak and Tang [1] that threshold models, based on cellular automata (CA), reproduce the Gutenberg-Richter power-law distribution of energy released during earthquakes, triggered considerable interest in simple models of earthquake dynamics. These CA models describe the discrete dynamics of a system that evolves to a scale-invariant (in space and time) statistical steady state. Such generic scale invariance has been coined self-organized criticality (SOC) [2].

An alternative approach to earthquake dynamics is based on a slider-block model, initially proposed by Burridge and Knopoff (BK) [3] and reintroduced by Carlson and Langer (CL) [4]. Since then numerous slider-block models have been studied; cf. the extensive reference list in Ref. [5]. The standard BK model describes the dynamics of a slowly driven slider-block chain in the presence of a nonlinear friction exhibiting a velocity softening instability. Nakanishi [6], trying to bridge these apparently distinct approaches, proposed a CA version of the CL model, whereas several attempts have been made to provide a continuum description of SOC [7].

Numerical solutions of BK-like models have shown that the system eventually settles into a periodic state of system-size earthquakes [8,9]. Moreover, the use of an artificial acceleration term to trigger the dynamics has been questioned [9,10]; its introduction in numerical simulations is a consequence of the choice of the dynamic friction force (no creep region at low velocities). The absence of a creep region has two other consequences: the working point of the system as determined by the imposed driving (tectonic) velocity is in the velocity softening region of the friction force, and it necessitates the use of a multivalued friction force at zero sliding velocity. These criticisms have led to questioning the relevance of slider-block models and the associated SOC models to earthquake dynamics.

Since the dynamic friction force is an essential ingredient of all BK-like models, we recently proposed a generalization (henceforth referred to as creep-slip model) of the original BK model that renders the phenomenological description of friction consistent [11]. Specifically, fault creep is taken into account by introducing an additional internal variable; under the imposed tectonic driving, the fault responds by a combination of rigid translation (sliding) and plastic displacement (irreversible deformation of some boundary layer of the fault). Herein we show numerically that the introduction of plastic deformation (and hence the introduction of a characteristic time scale of fault aging due to accommodation of asperities) in the original BK model removes some of the previously mentioned criticisms, and the simulations provide insight into the associated scaling and intermittency in one-dimensional dynamical systems.

In Ref. [11] a set of coupled partial differential equations for the shear forces (plastic deformation rates) and the sliding rates were derived. In this work we analyze numerically the dynamics of the corresponding slider-block model. For a one-dimensional discrete array of N_0 blocks, in a spatially discretized form (the second order gradient was discretized by the usual cental, three-point finite difference formula) and, expressed in dimensionless form, the dynamics is described by $2N_0$ coupled ordinary differential equations

$$\epsilon^2\partial_t e_k=f_k-\phi[\overline{v}(e_k+g_k)], \tag{1a}$$

$$\partial_t f_k=K(e_{k+1}+e_{k-1}-2e_k)-(e_k+g_k)+1, \tag{1b}$$

for $k=2,\ldots,N_0-1$, and free boundary conditions $k=1,N_0$. Here e_k is the dimensionless sliding rate of block k, f_k the shear force, g_k the plastic deformation rate, and ϕ the dynamic friction force. Apart from the stiffness parameter K (which arises from the spatial discretization) and the parameters that define the friction force, the model equations depend on $\overline{v}$, the tectonic drift velocity, and ϵ. The latter, expressed in terms of the original parameters is the ratio of the natural frequency of transversal oscillations of individual blocks to the plastic relaxation time; it is a measure of (temporal) stiffness of the system. The plastic deformation rate is related to the shear force f_k and the yield force f_y according to the (assumed) linear constitutive law

$$g_k=\begin{cases}\mathrm{sgn}(f_k)(|f_k|-f_y) & \text{for} \quad |f_k|>f_y\\ 0 & \text{otherwise.}\end{cases} \tag{2}$$

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Equation (1a) expresses the balance of forces, and Eq. (1b) expresses the time evolution of the driving force; the first term arises from compression/tension, the second from elastic shear, the third is the (assumed) linear force relaxation, and the last is the external driving. The original BK model [3] is recovered by taking f_y larger than the maximum friction force. In this case Eqs. (1) become identical to those reported in Ref. [12].

The discretized equations approximate the original continuum equations if the characteristic length introduced by the discretization, $K^{-1/2}$, is less than the smallest rupture lengths of interest. However, excessive computation times render the continuum limit ($K\to\infty$) difficult to attain.

The introduction of plastic deformation (and the corresponding time scale of fault aging) justifies a three-stage dynamic friction law $\phi(x)$: (a) at very low displacement velocities fault accommodation is fast enough to maintain optimum contact surfaces (velocity strengthening); (b) at intermediate velocities fault accommodation is incomplete because the fault aging rate is comparable to the displacement rate (velocity softening); and (c) at high velocities aging becomes negligible and inertia dominates (velocity strengthening, again). For the simulations these three stages were approximated as follows: (a) a linear velocity strengthening region, i.e., a region of positive velocity sensitivity; (b) a region of negative velocity sensitivity (inverse decay); and (c) an inertia regime ($x\to\infty$) with linear positive velocity sensitivity, again. For the precise functional form cf. Ref. [13]

We simulated numerically the dynamics of the chains of $N_0=16,\ 40,\ 100$, and 250 blocks. The numerical scheme is based on a variable-order, variable-step method implementing the backward differentiation formula. The results to be reported refer to systems operating in the velocity strengthening region (a), a choice suggested by extrapolation of laboratory friction data to actual tectonic drift velocities [14]. Moreover, the dynamics in the strengthening region (a) is qualitatively different from the well-studied dynamics in the softening region (b). In particular, if in the strengthening region the sliding rate (considered to be a fast variable, $\epsilon\to 0$) is adiabatically eliminated in favor of the shear force (friction dominates over inertia), the dynamics is determined by the time evolution of the shear forces, ($k=2,\ldots,N_0-1$),

$$\partial_t f_k = D(f_{k+1}+f_{k-1}-2f_k) - \frac{f_k}{f_{ss}} + 1 + \partial_t f_k|_{top}, \qquad (3)$$

where the diffusion coefficient is $D=K(1/f_{ss}-1)$, f_{ss} the steady-state force and $\partial_t f_k|_{top}$ is the corresponding toppling rule (force redistribution). For simplicity, we set $f_y=0$ in deriving Eq. (3). Equation (3) has a *short-wavelength instability*, which leads to uphill diffusion; for $f_{ss}>1$, the diffusion coefficient becomes negative. This is a consequence of the competition between stress release by rigid sliding and stress relaxation by plastic deformation; physically, it has been attributed to microfissuration [11]. Our choice of simulation parameters ensures that the working point is in this unstable region. Consequently, events do not have to be triggered artificially; any small perturbation is amplified. Note that this instability is absent in the original BK model (hence,

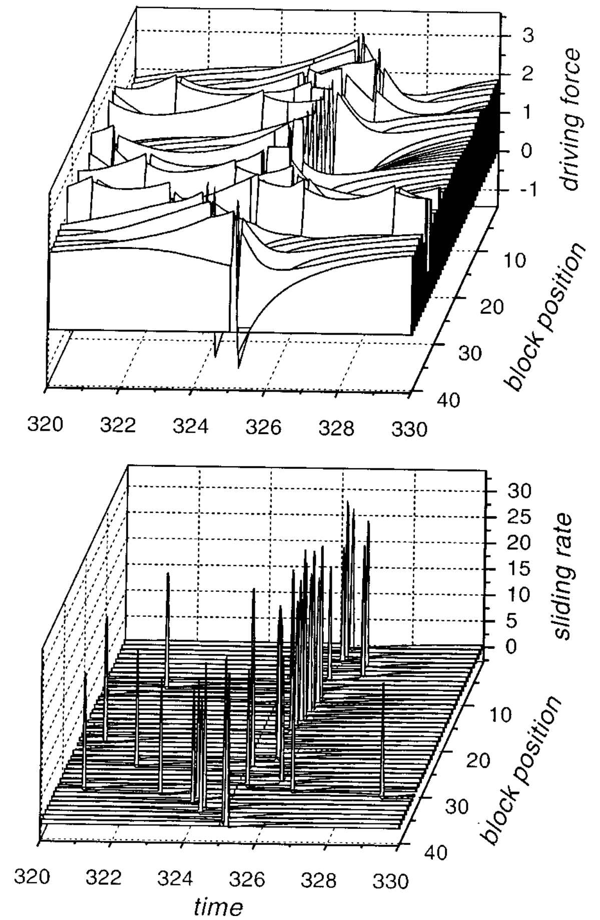

FIG. 1. Driving forces (top) and sliding rates (bottom) for a 40-block chain. The parameters are $\epsilon=0.05$, $f_{max}=3$, $f_y=2$, $\bar{v}=0.15$, and $K=1$.

it has not been studied before) and randomness is only introduced via the initial conditions, Eqs. (1) being deterministic.

Figure 1 shows the driving forces [Fig. 1(a)] and the sliding rates [Fig. 1(b)] as a function of time for each block in a 40-block system. The sliding rates exhibit strongly intermittent behavior: quiescent periods of almost negligible kinetic energy alternate with active periods. The latter involves either the failure of a single block or many-block events that appear as rupture fronts propagating along the chain.

We investigate spatial correlations by defining the event size as the total sliding displacement during a rupture, namely the time integral of the sliding rates over the duration of an event. The corresponding event-size probability density for a different number of blocks is plotted in Fig. 2. Note that for the larger systems the scaling region extends over two orders of magnitude, and the absence of a peak at the high end of the distribution (no excess of large events), as opposed to what was found with the original BK model. For small systems ($N_0=16$ and 40) single- and double-block events may be identified at the low end of the distribution. The scaling form of the event-size distribution arises from the dynamics of the system. The short-wavelength instability, a result of the introduction of plastic deformation, is essential in determining the spatial distribution of critical blocks (which fail if they encounter an event) and stable blocks (which can absorb the shock and thus stop an event). A scale-invariant distribution is obtained only when the spa-

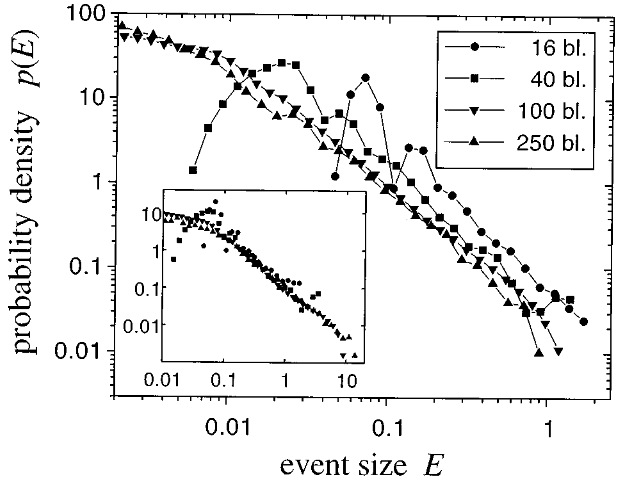

FIG. 2. Event-size probability density for chains of 16, 40, 100, and 250 blocks. The total number of events was approximately 50 000 (100 blocks). Inset: finite-size scaling plot.

tial stiffness (increasing stiffness forces the system to behave as a unit) and the force drop (as obtained from the friction law) are properly balanced: in that case the distribution does not exhibit a pronounced peak for large events.

Motivated by scaling arguments used in ordinary critical phenomena the event-size probability density is replotted assuming finite-size scaling, namely

$$p(E)dE \sim E^{-\beta}\exp(-E/N_0)dE. \tag{4}$$

The results are shown in the inset in Fig 2. The curves for different system sizes fall on the same universal curve with a scaling exponent $\beta = 1.5 \pm 0.1$, This is consistent with the exponent expected from the Gutenberg-Richter relation [15]. Moreover, the argument of the exponential cutoff shows scale invariance to be delimited only by finite-size effects.

Insight into the dynamics of the system is provided by studying temporal correlations. Figure 3 presents the power spectral density (PSD) of the intermittent time series generated by the spatially averaged sliding rate $[\overline{e(t)} = (1/N_0)\Sigma_i^{N_0} e_i(t)]$ for two different system sizes. At intermediate frequencies f the PSD scales as $S(f) \sim f^{-\sigma}$ with $\sigma = 1.25 \pm 0.1$. This behavior is a manifestation of ''flicker'' noise, i.e., $1/f^{\sigma}$ noise where σ is between 0 (uncorrelated white noise) and 2 (Brownian noise). In Fig. 3 the white noise behavior at low frequencies is a finite-size effect, whereas the high frequency peaks can be identified with signals arising from single-block events. In contrast, the PSD of the sliding rate of one typical block (out of N_0), inset in Fig 3, is dominated by white noise (flat spectrum without a scaling region).

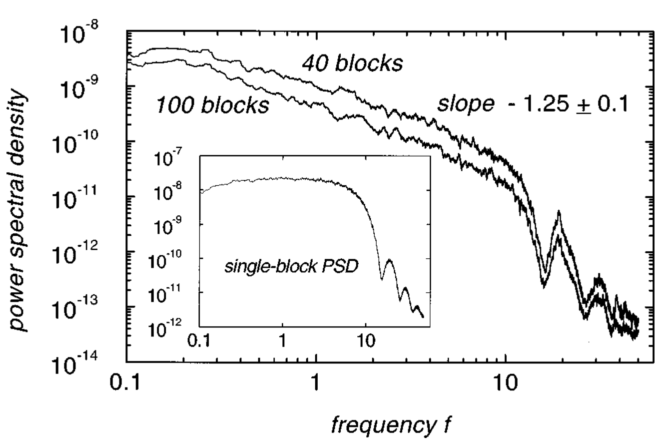

FIG. 3. Power spectral density (PSD) of the average-sliding-rate time series for a 40- and a 100-block chain. The inset shows the PSD of a single-block sliding rate (40-block chain).

The scaling behavior of the PSD may be understood by idealizing the time series as a sequence of quiescent and active intervals. The signal during the quiescent intervals, of duration $t_{\rm off}$, is set to zero, whereas the signal during the active intervals, of duration $t_{\rm on}$, is approximated by $\overline{e(t_{\rm on})} \sim t_{\rm on}^{\gamma}$. Note that $\gamma = 0$ corresponds to idealizing the time series by a binary series. The (anomalous) growth exponent γ is determined from the average event size for a given duration. We found numerically $\langle E(t_{\rm on})\rangle \sim t_{\rm on}^{1+\gamma}$ with γ approximately 0.3. The probability distribution of the duration of active periods $p_{\rm on}$ was found to scale as $\sim t_{\rm on}^{-\alpha}$ with $\alpha \approx 1.8$, and the probability distribution of the quiescent periods $p_{\rm off}$ was found to behave, for large intermission times $t_{\rm off}$, as $\sim \exp(-t_{\rm off}/\tau)$, where τ depends on the system size. A detailed analysis [13] of the joint probability distribution of event size and event duration leads to the scaling relation $\alpha = \beta + \gamma$.

Previous investigations [16] of intermittency in one-dimensional maps attributed the low frequency PSD divergence to the probability distribution of ordered intervals (''laminar'' in their terminology) under the assumption that the disordered intervals (''chaotic'' or ''turbulent'' bursts) were of zero duration. A generalization of their analysis to a sequence of quiescent (''disordered'') and active (''ordered'') intervals of finite duration with the previously described probability distributions and strengths [13] gives the scaling relation $\sigma = 1 + \gamma$ for $1 < \alpha < 2$. The numerically determined exponents satisfy this scaling relation.

The different statistics of quiescent and active intervals may be rationalized as follows: quiescent intervals are exponentially distributed because they may be broken up by an active interval (with a finite probability), whereas active intervals (being highly correlated) are only influenced by the system's state at the edge of the propagating rupture [17]. A similar conclusion was reached in an analysis of intermittency (laminar-to-turbulence transition) in terms of a one-dimensional array of *diffusive* coupled map lattices, where the statistics of the ordered (laminar) domains were algebraic, while the disordered (turbulent) domains were exponential [17].

The long-time (exponential) behavior of $p_{\rm off}$ implies that large events are statistically independent and their occurrence follows Poisson statistics. Thus, the probability of occurrence of a large event in the model-generated time series is independent of its history. This observation is corroborated by the flat PSD of a single block: local measurements are uncorrelated (in time); see, also, Ref. [18] for a discussion on earthquake predictability. The origin of the loss of temporal correlations is the short-wavelenght instability, which destroys such correlations (memory) during the quiescent periods. However, the short-time behavior of $p_{\rm off}$ was found to decay algebraically, a scaling law that is reminiscent of Omori's law for the power-law decay of aftershocks.

The results shown in Figs. 2 and 3 imply that the dynamics of the system as described by the discretized equations (1) may be cast in terms of a CA algorithm similar to those used to generate SOC states. In fact, Bak *et al.* [2] identified $1/f$ noise as the hallmark of temporal correlations in SOC systems. This algorithm becomes apparent when the toppling rule in the equation that determines the time evolution of the shear forces [Eq. (3)] is specified. Numerical analysis of the dependence of the *k*-block shear force f_k, on the sliding rate e_k, as described by Eqs. (1), suggests the following toppling rule: when the shear force on block *k* exceeds $f_{\max}$ (the friction force maximum) it is redistributed according to

$$f_k \rightarrow f_k - (f_{\max} - f_{\min}), \tag{5a}$$

$$f_{k\pm 1} \rightarrow f_{k\pm 1} + \frac{\Delta}{2}(f_{\max} - f_{\min}), \tag{5b}$$

where Δ is the ratio of distributed to released forces, $\Delta = 2K/(2K+1)$, and $f_{\min}$ can be related to the dynamic friction force ϕ [13]. Hence, force redistribution is dissipative, becoming conservative in the continuum limit, $K \rightarrow \infty$. Note however, two important differences with previous CA models [19] used to describe SOC states: (i) For $f < f_{\max}$, the dynamics is determined both by the external driving and, more importantly, by the diffusive coupling terms that give rise to *uphill diffusion*; (ii) a slow, but finite, time scale (associated with the deterministic evolution of the forces) is retained, the fast time scale having been adiabatically eliminated.

We simulated the one-dimensional diffusive CA model with parameters $(f_{\min}, f_{\max})$ determined from the solution of Eqs. 1 $(K=1)$. We found that the CA model also exhibits scaling; the event-size distribution scales with an exponent $\beta = 1.5$, in agreement with the results shown in Fig. 2. Preliminary results agree with our earlier observation that (spatial) scale invariance occurs when a proper balance of force redistribution (choice of $f_{\min}$ and $f_{\max}$) and force propagation (spatial stiffness K) is reached.

In summary, the slider-block realization of the creep-slip model with velocity-softening instability and the associated diffusive CA algorithm with threshold dynamics have revealed a close relation between those apparently distinct concepts. For particular choices of parameters, both descriptions of the system evolve to a scale-invariant steady state. The existence of such a state shows the importance of uphill diffusion in driving the system. Moreover, the CA model is dissipative (for finite K) suggesting that, in one dimension, local force conservation is not a necessary requirement to obtain a scale-invariance steady state [20].

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